

# THE INTRICACIES OF TAXING BANKS \*

Arnoud W.A. Boot

University of Amsterdam and CEPR

email <a.w.a.boot@uva.nl>

Casper G. de Vries

WRR, Tinbergen Institute and

ESE Erasmus University Rotterdam,

Corresponding author, email <cdevries@ese.eur.nl>

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## Abstract

We investigate the incentives for asset risk taking by banks under alternative corporate tax regimes including ACE, TCR and bank levies. The literature has shown that levies linked to bank debt financing, capping interest deductions for corporate taxes or adding tax advantages to equity all discourage leverage. We show, however, that levies, like caps on interest deductions increase asset risk. Per contrast, a system that levels the playing field between modes of funding by providing equity with tax advantages reduces risk-taking incentives. Our results highlight that amendments to the corporate tax system should consider asset risk incentive considerations and not just incentives for leverage.

## 1 Introduction

How should banks be taxed? Many have argued that bank stability would be served by having banks be better capitalized, yet the corporate tax system continues to favor debt (De Mooij and Keen, 2016; IMF, 2016). Interest payments are tax deductible, but equity has not been granted a similar favorable treatment. While the impact of the tax system on leverage incentives has received considerable attention, the literature has not focused much on how banks' asset-risk choices are affected by the tax system. This is somewhat surprising as banks, more so perhaps than non-financial corporations, can rather easily change their asset risk profile, see Myers and Rajan (1998).

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Several amendments to the corporate tax regime have been proposed to address the preferential treatment of debt. Two main approaches can be distinguished: one seeks to limit the deductibility of interest payments – the Thin Capitalization Rule (TCR) – and the other aims at an equal treatment of debt and equity by granting deductibility for the cost of capital (the Allowance for Corporate Equity, ACE). Also observed are tax levies on the level of (un-)insured liabilities. All limit the favorable tax treatment of debt.<sup>1</sup> Our focus is on the impact of these measures on asset risk, holding leverage in most of our analysis fixed. Effectively this implies that leverage is constrained by capital regulation, and that this regulation is binding.<sup>2</sup>

Before discussing TCR and ACE, observe that levies and corporate tax are ways to generate revenues for the government. From the perspective of reducing risk taking incentives, a corporate tax is better: it reduces the after tax cash flow associated with risk taking. That is, when risk taking works out, the high realized cash flow is taxed. A bank levy on debt does not eat into high cash flow realizations. The rewards of risk taking are not taxed. For governments this offers a clear policy recommendation. The popularity of bank levies following the 2007-08 financial crisis (to punish ‘excessive’ risk taking) is misguided from an asset risk taking perspective.

Next, consider the TCR and ACE schemes. The question is, how do TCR and ACE compare when considering risk taking incentives for a bank?<sup>3</sup> Our key finding is that the effects on risk taking are very different between ACE and TCR. Capping the interest deductibility (TCR) worsens risk taking incentives: it tempts banks to increase their risk. Giving equity similar tax advantages as debt (ACE), has the opposite effect: risk taking is reduced. In other words, TCR leads to more risk taking (worsens moral hazard), while ACE reduces risk taking.<sup>4</sup> Note that we focus on risk taking incentives and we do not, however, claim to have a fully fledged general equilibrium model that could define a first best level of risk for society.

We do take into account that the loss in tax revenue associated with ACE

<sup>1</sup> Mostly levies are linked to debt financing. There are also examples where the levies are linked to the level of total assets (see Devereux, Johannessen and Vella, 2019).

<sup>2</sup> Observe that all tax measures we focus on, i.e., TCR, ACE and levies, do reduce the benefits of debt financing. Gresik et al. (2017), Brekke et al. (2017) and Juranek et al. (2018) discuss the effects of the ACE and TCR on leverage for non-financial firms. Hebous and Ruf (2017) study the effect of ACE for German based multinationals and find that corporate debt decreases following an ACE. Schepens (2016) shows empirically that the ACE regime could incentivize banks to take more equity in their capital structure. De Mooij, Keen and Orihara (2014) show empirically that corporate tax rate changes affect risk taking, but only have a small effect on the degree of leverage (especially so for the larger banks). The latter could be explained by the ratchet and debt overhang effects as highlighted by Admati, DeMarzo, Hellwig and Pfleiderer (2018). In a report to the Dutch government we discussed the TCR and ACE schemes in connection with the Dutch banking sector (WRR, 2019).

<sup>3</sup> Moral hazard (risk taking, asset substitution) in banking has been studied extensively, see for example Hellman, Murdock and Stiglitz (2000), Myers and Rajan (1998), and the textbook by Greenbaum, Thakor and Boot (2019).

<sup>4</sup> Roe and Troge (2016) propose to apply the ACE only to equity in excess of a (to be determined) minimum level. This would help level the tax treatment at the margin and limit the loss in tax revenues.

can be mitigated with an increase in the corporate tax rate. This reinforces results of a higher corporate tax rate on risk taking. But as TCR comes with extra tax revenues due to the cap, the corporate tax rate has to be lowered. Thus TCR encourages risk taking instead. Overall, we find therefore that ACE leads to less risk taking and TCR to more.<sup>5</sup>

An extreme version of the TCR is the Comprehensive Business Income Tax (CBIT) that allows zero deductibility which is the most severe cap possible. The allowed amount of interest deduction on debt financing then becomes zero. From a leverage perspective this is fine: it creates an equal tax treatment (equivalence) between debt and equity. From a risk taking perspective, however, the lower corporate tax rate that comes with CBIT worsens risk taking incentives.

The other extreme regime is the Allowance for Corporate Capital (ACC) that fully equates debt and equity by imputing a 'normal return' on all assets regardless the way these are financed.<sup>6</sup> This normal return is deductible before tax as a business expense. The corporate tax rate then needs to increase, which is good for containing risk taking incentives.

A final question we ask is, if the government wants to increase its revenues without affecting bank risk taking what options does it have? Note that nor an increase in tax rates nor changing levies can be done without having an impact on risk taking (incentives). We show that the government can increase its revenues without affecting risk if it uses the TCR or ACE together with -in both cases- an adjustment in the corporate tax rate.

The last section considers the case in which the capital requirements are still binding but risk-based (i.e. in the spirit of Basel III). We show that the results carry over to this setting.

The organization of the paper is as follows. Section 2 discusses the model setup, and includes the analysis of bank levies. The analysis of the ACE and TCR follows in section 3. In section 4 we consider risk-based capital requirements. Section 5 concludes.

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<sup>5</sup>Several refinements to ACE and TCR have been analyzed and both schemes have been introduced in several jurisdictions. Belgium and Italy are two well known cases where an ACE was implemented, though more recently these have been (partially) phased out. Schepens (2016) finds that the introduction of ACE in Belgium increased equity ratios and reduced risk for low capitalized banks. Italy had a system where the ACE only applied to the net increases in equity (see Branzoli and Calumi, 2018). The alteration and later reversal in Belgium is in part due to problems with having a different tax structure compared to other EU-countries, in conjunction with the alternative route of lower corporate tax rates that most countries have chosen (De Mooij, Hebois and Hrdinkova, 2018).

A number of other countries follow the TCR approach, i.e. limit the interest deductibility on debt via a cap on the interest expense that can be deducted. Italy has both a cap and a (limited) ACE, see Branzoli and Calumi (2018).

Bank levies are present in many countries. Following the 2007-08 global financial crisis, in part on instigation of the IMF (see Devereux, Johannesen and Vella, 2019; IMF, 2010), several countries introduced bank levies. More recently, bank levies have again gained prominence particularly as an instrument to combat 'excessive' profits in banking.

<sup>6</sup>Belgium had an ACE resembling the ACC.

## 2 Model setup

We build on the Monti-Klein model in which banks choose the level of asset risk. This enables us to investigate the effect on risk taking due to bank levies, ACE and TCR in a setting with corporate taxes and interest deductibility. The Monti-Klein model allows banks to earn rents, and thus takes us away from a purely competitive model; see for an extensive discussion of the Monti-Klein model, Freixas and Rochet (1997, section 3.2.).

We consider a one-period economy where at time zero a bank chooses the riskiness of its assets (loans), and at time one loans mature and the bank liquidates with all proceeds distributed to its financiers – equity holders and depositors. The bank maximizes the value to equity holders.

The payoffs of the bank are dichotomous, either a bank does well on its loan book, or it fails. The success probability is  $p(s)$ . If successful, the loans pay a return  $s$ , which together with the full principal – equal to 1 – can be paid to depositors and equity holders. The payment for deposit insurance is subsumed under the costs  $c$  of intermediation.<sup>7</sup> With probability  $1 - p(s)$  the loans fail and everything is lost. Depositors can recoup their losses through the deposit insurance fund for which the bank pays a premium  $c$ .<sup>8</sup>

The choice variable  $s$  can be seen as the risk choice of the bank, determining both the success probability  $p(s)$  and the return  $s$  in the good state. The success probability is a continuous function in  $s$ , such that  $p'(s) < 0$ . Thus, projects with a higher return  $s$ , are less likely to be successful. Hence,  $p(s)$  is a decumulative probability function, that is if  $F(s)$  is the distribution function of failures, then its complement  $p(s) = 1 - F(s)$  is the probability of success.

In the analysis we need the following condition:

**Condition 1** *The success probability  $p(s)$  is a twice differentiable decumulative distribution function with the following property*

$$\Gamma(s) \equiv -\frac{p(s)p''(s)}{p'(s)} + 2p'(s) < 0 \quad (1)$$

This property guarantees that the second order condition for optimality is satisfied. For example it is easy to check that this condition holds for the exponential distribution. How restrictive is this condition? First note that popular conditions like concavity or log-concavity are more restrictive. For example, consider the beta distributions  $F(s) = s^\beta$  on  $[0, 1]$ . For  $\beta > 1$ , the condition is satisfied throughout the support. But for  $\beta < 1$  when the success probability  $p(s)$  is convex, the condition still holds for the larger values of  $s$ . Nevertheless, all these beta distributions imply that  $p(s)$  is log-concave. But then again, Pareto distributions with a finite mean do satisfy (1), but imply that  $p(s)$  is log-convex. Thus the Condition 1 essentially is a requirement on the success probability in the tail area. The shape of a distribution in the tail

<sup>7</sup>Phillipon (2015) provides evidence that the cost function is essentially linear.

<sup>8</sup>The deposit insurance fund levies a fair premium, hence the premium is endogenous. But the analysis shows that a fixed premium would not change the results materially.

area is related to how the distribution of the (linearly rescaled) maximum of  $n$  independent draws from  $F(s)$  behaves. Extreme value distributions are the limiting distributions of the maximum as the sample size  $n$  increases.

It turns out that the Condition 1 is implied by the much weaker assumption that the distribution is in the max-domain of attraction of any one of the three extreme value distributions, provided that the distribution is twice differentiable and has a finite mean. This is shown formally in Appendix A. The condition therefore comprises most of the standard distributions like the normal, exponential, Pareto, Student-t, F-distribution, uniform, beta, etc. One note of caution is that while the condition certainly applies in the tail area, it may not hold globally for a specific distribution that is in the domain of attraction of an extreme value distribution, but then again we only need (1) for  $s \geq 0$  and mostly deeper into the right tail area.

We can immediately make use of the Condition 1 to determine the properties of a fair insurance premium.<sup>9</sup> Let  $D$  denote the amount of deposits. The probability of default is  $1 - p(s)$  and in that case the insurance fund needs to cover the deposits plus interest

$$(i + 1)D$$

i.e., the principal of the depositors plus lost interest. With probability  $p(s)$  the bank does well and the fund collects the premium. A fair insurance premium  $c(s)$  then requires

$$p(s)c(s) = (1 - p(s))(i + 1)D$$

So that

$$c(s) = \left( \frac{1 - p(s)}{p(s)} \right) (i + 1)D$$

Note that a marginal increase in  $s$  then requires

$$c'(s) = -\frac{p'(s)}{p(s)^2} (i + 1)D > 0 \quad (2)$$

to keep the fund solvent. Furthermore

$$\begin{aligned} c''(s) &= \left( -\frac{p''(s)}{p(s)^2} + 2\frac{(p'(s))^2}{p(s)^3} \right) (i + 1)D \\ &= \frac{-p(s)p''(s) + 2p'(s)^2}{p(s)^3} (i + 1)D \\ &= \frac{p'(s)}{p(s)^3} \Gamma(s) (i + 1)D > 0 \end{aligned} \quad (3)$$

Under the condition in (1),  $\Gamma(s) < 0$  and hence the fair premium is a convex function in the risk choice  $s$  by the bank.

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<sup>9</sup>See Dermine (1986), Chan, Greenbaum and Thakor (1992) and Freixas and Rochet (1998) for a discussion of the pricing of deposit insurance.

## 2.1 A bank's balance sheet

Consider a simplified balance sheet of a limited liability bank. On the asset side there are loans, which we normalize to 1. The bank is financed by equity  $k$  and deposits are  $D = 1 - k$ . The bank pays depositors an interest rate  $i$ . As deposit insurance makes deposits essentially risk free, we take  $i$  fixed. The bank is a price taker in the deposit market. Furthermore, equity holders require a return  $R$  on their investment. We take this required return as exogenous as well. The corporate income tax rate is denoted by  $t$ . The bank pays a fair insurance premium  $c(s)$ .<sup>10</sup> Observe that in the good state the owners hold on to their capital investment and make an after tax income at time one equal to

$$(1 - t)[s - iD - c(s)] + k$$

We can state the objective function as follows (recall  $D = 1 - k$ )

$$\max_s p(s) \{(1 - t)[s - i(1 - k) - c(s)] + k\} \frac{1}{1 + R} - k \quad (4)$$

Note that a necessary condition for the owners of the bank to participate is that at the optimum  $s$ , expected operating income should be positive, i.e.,

$$p(s)[s - i(1 - k) - c(s)] > 0$$

Otherwise the owners can not be expected to make a positive NPV. The First Order Condition (FOC) reads

$$p'(s)\{(1 - t)[s - i(1 - k) - c(s)] + k\} + p(s)(1 - t)(1 - c'(s)) = 0 \quad (5)$$

From this we get

$$\{(1 - t)[s - i(1 - k) - c(s)] + k\} = -\frac{p(s)(1 - t)(1 - c'(s))}{p'(s)} \quad (6)$$

Note that the left hand side of (6) is positive. Hence, given that  $p'(s) < 0$ , it must be the case that  $c'(s) \in (0, 1)$ . This fact is used below in the analysis of the tax regimes ACE and TCR. The equality (6) is also used to simplify the Second Order Condition (SOC).

The second order sufficient condition reads

$$\begin{aligned} & p''(s)\{(1 - t)[s - i(1 - k) - c(s)] + k\} \\ & + 2(1 - t)p'(s)(1 - c'(s)) - p(s)(1 - t)c''(s) \\ & = (1 - t) \left[ -\frac{p(s)p''(s)}{p'(s)} + 2p'(s) \right] (1 - c'(s)) - p(s)(1 - t)c''(s) \\ & = (1 - t) \left[ \Gamma(s) \left( 1 + \frac{p'(s)}{p(s)^2} (i + 1)(1 - k) \right) - \frac{p'(s)}{p(s)^2} \Gamma(s)(i + 1)(1 - k) \right] \\ & = (1 - t)\Gamma(s) \end{aligned} \quad (7)$$

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<sup>10</sup>For simplicity we ignore other costs.

Where we made use of the FOC (5) to simplify the SOC and the properties of the fair premium in (2) and (3), recalling  $D = 1 - k$ . So it suffices that the Condition 1 applies.

How does a bank react to an increase in the leverage condition? Consider the FOC (5) and totally differentiate (5) with respect to capital  $k$  and its risk  $s$ :

$$(1 - t) \Gamma(s) ds + p'(s) [(1 - t)i + 1] dk = 0$$

The model then captures that more capital reduces risk:

$$\frac{ds}{dk} = -\frac{p'(s) [(1 - t)i + 1]}{(1 - t) \Gamma(s)} < 0 \quad (8)$$

since both  $\Gamma(s) < 0$  and  $p'(s) < 0$ . But given positive externalities due to e.g. the maintenance of the payment system, a 100% capital rule would be suboptimal, so that we take  $k \in (0, 1)$ . As stated before, we let capital regulation be binding, thus  $k$  is fixed. As we do not have a full general equilibrium model, our analysis does not address the optimal level of risk  $s$  (and  $k$ ) for society, nor the potential liquidity benefits (including payment system externalities) of having banks be financed in part with deposits.

## 2.2 The embedded moral hazard

Differentiate the FOC (5) with respect to  $s$  and  $i$ :

$$(1 - t) \Gamma(s) ds - p'(s) (1 - t) (1 - k) di = 0$$

This gives the embedded moral hazard problem:

**Lemma 1** *If Condition 1 applies, then*

$$\frac{ds}{di} = \frac{p'(s)}{\Gamma(s)} (1 - k) > 0 \quad (9)$$

At higher interest rates on deposits, the bank chooses to finance projects that are more risky. Typically, small depositors cannot gauge well the risks embedded in the loan book of the bank and may not even have incentives to discipline bank behavior in the presence of deposit insurance. The effect signifies the room for moral hazard that banks have.

In a similar way, one obtains the moderating effect that a corporate tax hike has on risk taking. To this end differentiate the FOC with respect to  $s$  and  $t$ , we get

$$(1 - t) \Gamma(s) ds - \{p'(s) [s - i(1 - k) - c(s)] + p(s) (1 - c'(s))\} dt = 0$$

and use that the FOC (5) expressed differently reads

$$p'(s) [s - i(1 - k) - c(s)] + p(s) (1 - c'(s)) = \frac{-kp'(s)}{(1 - t)}$$

This gives the next lemma:

**Lemma 2** *An increase in the corporate tax rate lowers risk taking since*

$$\frac{ds}{dt} = -\frac{kp'(s)}{(1-t)^2 \Gamma(s)} < 0 \quad (10)$$

The increase in the tax rate reduces the benefits of risk taking by lowering the payoff in the good state, while the payoffs in the bad state do not change. This induces less risk taking to reduce the likelihood of a bad outcome (bank owners loosing their capital).<sup>11</sup>

### 2.3 Bank levies

In the aftermath of the credit crisis several countries instituted a bank levy. A bank levy is a specific tax on deposits, see Devereux et al. (2019).<sup>12</sup> Within the setup of the model with only deposits, a levy  $\lambda$  can be introduced as follows

$$\max_s p(s) \{(1-t)[s - i(1-k) - \lambda(1-k) - c(s)] + k\} \frac{1}{1+R} - k \quad (11)$$

Devereux et al. (2019) find in their analysis of European banks that a bank levy reduced funding risk, but also increased portfolio risk. This contrasts with the mitigating effect of increasing the corporate tax rate on risk taking. Our theoretical model corroborates their findings:

**Proposition 1** *An increase in the bank levy rate  $\lambda$  increases bank risk, since*

$$\frac{ds}{d\lambda} = \frac{p'(s)}{\Gamma(s)} (1-k) > 0 \quad (12)$$

The proof is relegated to Appendix B. The increased asset risk (12) can be explained from the moral hazard effect (9). Just as an increase in the deposit rate  $i$  makes a bank to take on more risk, a bank levy triggers a bank to look for loans with a higher payoff to recoup the cost of the bank levy in the good state. Note the contrast with increasing the corporate tax rate  $t$ . As we showed in (10), increasing the corporate tax rate lowers risk taking. From a perspective favoring lower risk, a corporate tax increase clearly dominates bank levies.<sup>13</sup>

Next we look into the ACE and TCR systems.

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<sup>11</sup>In fact, from a (containing) risk taking perspective, a progressive corporate tax rate may help in this setting. That would discourage risk taking even more (if desirable). Moreover, it could for other reasons have benefits, e.g. increasing bank size (and consolidation) would be discouraged as it could lead to a more than proportional increase in corporate taxes. Considering concerns about too-big-to-fail institutions this might be desirable.

<sup>12</sup>Célèrier et al. (2020) study and analyze empirically the effects of a bank levy on unsecured debt.

<sup>13</sup>Recently, Italy has approved an excess profit tax which applies to 2023 profits exceeding profits in earlier years above a certain threshold (EY, 2023). Such tax has better risk taking characteristics than a bank levy as defined in this section.

### 3 Analysis of ACE and TCR

We start with examining the impact that the ACE or the alternative TCR regime have on risk taking.

#### 3.1 ACE

In the ACE regime there is an additional deduction from earnings before taxes apart from the interest on deposits. This is the deduction applied to the so called notional return on equity. Let  $g$  denote the notional return on equity. The deduction from taxes is imputed as a tax refund on capital  $tgk$  in the good state. The objective function of the bank becomes (cf., (4)),

$$\max_s p(s) \{(1-t)[s - i(1-k) - c(s)] + k + tgk\} \frac{1}{1+R} - k \quad (13)$$

The FOC reads

$$p'(s) \{(1-t)[s - i(1-k) - c(s)] + (1+tg)k\} + (1-t)p(s)[1 - c'(s)] = 0 \quad (14)$$

This implies that at the optimum

$$(s - i(1-k) - c(s)) + \frac{1+tg}{1-t}k = -\frac{p(s)}{p'(s)}[1 - c'(s)] \quad (15)$$

It is straightforward to check that the second order sufficient condition again only requires  $\Gamma(s) < 0$ , see Condition 1, and that the positive left hand side of (15) implies that  $c'(s) < 1$ .

It is then straightforward to establish the effect of introducing an ACE ( $g > 0$ ) on risk taking. We have:

**Proposition 2** *Introducing ACE ( $g > 0$ ) reduces risk taking. That is,*

$$\frac{ds}{dg} = \frac{-kp'(s)}{\Gamma(s)} \frac{t}{(1-t)} < 0$$

**Proof.** Differentiate the FOC (14) with respect to  $s$  and  $g$ :

$$\begin{aligned} & p''(s) \{(1-t)[s - i(1-k) - c(s)] + (1+tg)k\} ds \\ & + 2(1-t)p'(s)[1 - c'(s)] ds - (1-t)p(s)c''(s)ds + p'(s)tkdg \\ = & (1-t) \left\{ -\frac{p''(s)}{p'(s)}p(s)[1 - c'(s)] + 2p'(s)[1 - c'(s)] - p(s)c''(s) \right\} ds \\ & + p'(s)tkdg \end{aligned}$$

where we use (14) in the second step. Subsequently, use the SOC (7) to simplify this further

$$(1-t)\Gamma(s)ds + p'(s)tkdg$$

Then equate this to zero and solve for  $ds/dg$ . ■

The result in Proposition 2 establishes that the introducing ACE reduces risk taking incentives.

The introduction of the ACE lowers the government revenue, this can be compensated by raising the tax rate. Similar to Lemma 2 one shows that given a certain level of ACE:

**Corollary 1** *At a given level of  $g$ , raising the corporate tax rate lowers risk taking by*

$$\frac{ds}{dt} = \frac{-kp'(s)}{(1-t)^2 \Gamma(s)} (1+g) < 0$$

**Proof.** Similar to the proof of Proposition 2, but differentiate (14) with respect to  $s$  and  $t$ . ■

Both the Proposition 2 and the Corollary 1 imply that the level of risk taking is reduced.

A combination of the ACE and tax hike affects the profitability of banks. To secure support for the introduction of the ACE policy, a question is whether it is possible to reduce risk taking while leaving expected profits of the banks fixed (this also makes the ACE better comparable to the TCR regime below). To address this question we make use of the envelope theorem. Note that there is one variable  $s$  that the bank uses to maximize its profits and two government policy parameters  $t$  and  $g$  that the bank takes as given. For each combination of  $g$  and  $t$ , let  $s(t, g)$  denote the (expected) profit maximizing choice of  $s$  in (13). Write the resulting maximized bank profits as

$$\Pi(s(t, g); t, g)$$

Consider marginal changes in the two policy variables and how these affect profits  $\Pi(s(t, g); t, g)$ . By the envelope theorem we only need to consider the direct effects of (marginal) changes in  $t$  and  $g$ . This gives:

$$\begin{aligned} \frac{d\Pi(s(t, g); t, g)}{dt} &= \frac{\partial\Pi(s(t, g); t, g)}{\partial t} \\ &= \frac{p(s(t, g))}{1+R} \{ -[s(t, g) - i(1-k) - c(s(t, g))] + gk \} \end{aligned}$$

and

$$\frac{d\Pi(s(t, g); t, g)}{dg} = \frac{\partial\Pi(s(t, g); t, g)}{\partial g} = \frac{p(s(t, g))}{1+R} tk$$

Subsequently, impose that the ACE intervention leaves expected profits unchanged. Hence, we must have that

$$d\Pi(s(t, g); t, g) = \frac{\partial\Pi(s(t, g); t, g)}{\partial t} dt + \frac{\partial\Pi(s(t, g); t, g)}{\partial g} dg = 0$$

Rearranging, this yields the requirement that policy changes must be such that<sup>14</sup>

$$\frac{dt}{dg} = \frac{kt}{[s - i(1 - k) - c(s)] - kg} \quad (16)$$

We are ready to investigate the effects of introducing a notional return on equity accompanied by a change in the corporate income tax rate such that expected bank profits are not affected, i.e., such that the change in the tax bill is neutral for the bank. To this end differentiate the FOC (14) totally with respect to  $s$ ,  $g$  and  $t$  and use (15) as well as the condition (1) to simplify the resulting expression

$$\begin{aligned} & \Gamma(s)(1-t)ds \\ = & -p'(s)ktdg + [p'(s)\{[s - i(1 - k) - c(s)] - kg\} + p(s)(1 - c'(s))]dt \end{aligned} \quad (17)$$

Subsequently, impose profit neutrality by substituting (16) into (17)

$$\begin{aligned} \Gamma(s)(1-t)ds &= -p'(s)kt \frac{[s - i(1 - k) - c(s)] - kg}{kt} dt \\ &+ [p'(s)\{[s - i(1 - k) - c(s)] - kg\} + p(s)(1 - c'(s))]dt \end{aligned}$$

By cancelling terms on the right hand side, we obtain:

**Proposition 3 (ACE regime)** *The introduction of a notional return deductibility on equity together with an adjustment in the tax rate  $t$  such that policy changes do not affect expected bank's profits, reduces risk taking incentives, since*

$$\left. \frac{ds}{dt} \right|_{\text{profits}} = \frac{p(s)[1 - c'(s)]}{(1 - t)\Gamma(s)} < 0 \quad (18)$$

The introduction of the deductible  $gk$  comes with an increase in the tax rate  $t$  to compensate for the extra deductible. Both  $t$  and  $g$  reduce risk taking  $s$ , but are combined in such a way that expected profits of the bank are unaffected.<sup>15</sup>

The Proposition 2 and the Corollary 1 show that introducing an ACE has reinforcing favorable effects on risk taking incentives: both the equity deduction  $g$  and the effect of increasing the tax rate  $t$  reduce risk taking incentives. The intuition is that the equity deduction  $g$  enhances the value of the good state, inducing a lower risk choice to make it more likely that this benefit is obtained. The commensurate increase in the tax rate reduces the benefit of risk-taking (i.e. a higher proportion of  $s$  is taxed away), hence also this leads to less risk taking. The combined effect of the two instruments is thus positive as highlighted in Proposition 3. To conclude, imposing an ACE regime is commendable as it reduces risk taking while it can be implemented in such a way that it does not affect expected bank profits.

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<sup>14</sup>Note that at the introduction of the ACE policy, initially  $g = 0$ , so that  $dt/dg > 0$  definitely.

<sup>15</sup>Section 3.3 focuses on the case where expected government revenue is held constant.

### 3.1.1 ACC

A special case of the ACE constitutes the Allowance for Corporate Capital, ACC for short. The ACC regime imputes a “normal return”  $g$  on the total value of the bank’s assets. This notional return over all funding sources is tax deductible, and replaces the deductibility of interest payments only. As before, the tax rate is adjusted to ensure bank profits are not affected. The ACC implies the following objective function

$$\max_s p(s) \{(1-t)[s - c(s)] - i(1-k) + k + tg\} \frac{1}{1+R} - k \quad (19)$$

Observe that  $tg$  is the overall tax rebate on the total funding cost (debt plus equity). We can now obtain the following result (similar to Proposition 3):

**Corollary 2** *The introduction of an ACC while maintaining profits reduces risk taking incentives.*

**Proof.** See Appendix B. ■

### 3.2 TCR

The alternative regime is to cap the interest deduction and to compensate with a lower corporate tax rate so as to maintain bank expected profits. Consider the introduction of a binding constraint  $\beta$ ,  $\beta < 1$ , such that only  $\beta i(1-k)$  of the deposit interest expenses is tax deductible. We can restate the objective function in the case of the TCR as follows

$$\max_s p(s) \{(1-t)[s - c(s)] - i(1-k) + t\beta(1-k)i + k\} \frac{1}{1+R} - k \quad (20)$$

By differentiating the FOC with respect to  $s$  and  $\beta$ , see (39) Appendix B, one obtains the following partial effect:

**Lemma 3** *Increasing the cap, i.e., lowering  $\beta$ , increases risk taking incentives since*

$$\frac{ds}{d\beta} = -\frac{p'(s)(1-k)ti}{(1-t)\Gamma(s)} < 0 \quad (21)$$

**Proof.** See Appendix B. ■

The TCR policy can also be implemented in such a way as to maintain expected bank profits. This requires a simultaneous change in the corporate tax rate. We have the following:

**Proposition 4** *The introduction of the TCR regime, while maintaining bank profits constant, increases risk taking incentives since*

$$\frac{ds}{dt} \Big|_{\text{profits}} = \frac{p(s)[1 - c'(s)]}{(1-t)\Gamma(s)} < 0 \quad (22)$$

**Proof.** See Appendix B. ■

Hence, introducing the cap  $\beta < 1$  and lowering  $t$  in such a way as to maintain bank profitability unambiguously increases risk taking by the bank. The intuition is that the cap on the interest deductibility increases the (net) interest burden in the good state, and this can be reduced in expected value sense by increasing risk. The commensurate tax rate reduction also increases risk taking incentives as a smaller proportion of the returns  $s$  is taxed away.

Observe that both in the case of the ACE and the TCR the effect of a change in the tax rates go in the same direction, i.e. the derivatives (18) and (22) are identical. But in case of the TCR, the tax rate has to be lowered due to the cap and maintaining profits, while the higher deductible in the case of ACE requires an increase in the marginal tax rate. The difference between the two regimes is thus that in the case of TCR, both the cap  $\beta$  and the tax rate act in the same way on the deductible deposit interest rate  $t\beta(1 - k)i$ , but in the case of ACE this deductible  $t(1 - k)i$  is left unchanged while an extra deductible  $tgk$  is added.

### 3.2.1 CBIT

The comprehensive business income tax implies zero deductibles. The CBIT is a special case of the TCR, i.e., it sets  $\beta = 0$ . Note that a cap at zero requires that the initial corporate tax rate is lowered in order to preserve profits. We have the following result.

**Corollary 3** *The introduction of the CBIT increases risk taking incentives.*

The result of Corollary 3 immediately follows from Proposition 4. The CBIT is the most extreme version of the TCR.

### 3.2.2 Comparison of CBIT and ACC

We briefly compare the two extreme versions of the ACE and TCR. Note first that both ACC and CBIT regimes are neutral regarding the choice between debt and equity financing. But as Corollaries 2 and 3 show, the CBIT comes with a lower tax rate and hence worsens risk taking, while in the ACC regime the increased tax rate tempers risk taking.

## 3.3 Raising Tax Revenue

The question we address here is how to increase tax revenue without affecting the risk level  $s$ .<sup>16</sup> We consider three instruments: a bank levy, ACE and TCR.

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<sup>16</sup>We owe an anonymous referee for asking this interesting question.

### 3.3.1 Bank levy

First consider the bank levy  $\lambda$ . Increasing  $\lambda$  increases tax revenue, but Proposition 1 shows that this also increases the risk level  $s$ . The problem is that one needs at least two instruments with different effects of  $t$  and  $g$  on  $s$ , say, to be able to raise revenues yet keep risk the same. The ACE and TCR alternatives may therefore work as these involve two instruments.

### 3.3.2 ACE

Consider the ACE regime. In this case use the total differential in (17), but set  $ds = 0$ . This gives

$$\frac{dt}{dg} = -\frac{(1-t)t}{1+g} < 0$$

where we used (15) to simplify the expression. To keep the risk level fixed while increasing tax collection by increasing the corporate tax rate  $t$ , requires that the notional return on equity is lowered. This is because with  $s$  left free, we find from Proposition 2 and Corollary 1 that both  $ds/dg < 0$  and  $ds/dt < 0$  and thus go in the same direction. Therefore at  $g > 0$  tax collection is enhanced if  $t$  is increased while at the same time the budget is relieved by lowering  $g$  to keep  $s$  constant. Note that at  $g = 0$ , a further increase in  $t$  would even require a negative  $g$ . Thus this combination of instruments only makes sense if the ACE regime is already in place, and can only accommodate a limited increase in government revenues.

### 3.3.3 TCR

Turning to the TCR. The key insight now is that revenue not only increases with a tighter cap, but may also go up with the tax rate increase (if  $\beta$  is close to 1). In Appendix B we show that

$$\frac{d\beta}{dt} = \frac{(1-k)(1-\beta)i - k}{(1-k)(1-t)i}$$

This expression is negative if the cap is not very strong, i.e., for  $\beta$  close to 1. Thus, to raise the budget the TCR may have an advantage over the ACE in that it works if the cap is not yet in place yet, i.e., if  $\beta = 1$ , and  $\beta$  is slightly lowered. Also note that contrary to the ACE, the adjustment of the cap reinforces the revenue effect.

## 4 Risk based capital requirements

So far, capital requirements are not only binding but also of the simple Basel I variety: capital is in fixed proportion of lending (assets). We now allow for risk-based capital requirements, but adopt a fixed deposit insurance premium for simplicity and tractability of the analysis.

So far we have considered a capital requirement that forces banks to hold  $k$  of capital for each 1\$ lend. With risk-based capital requirements under the Basel III rules, a bank has to hold a minimum level of capital related to its risk profile, i.e.,  $k \geq \phi s$  say. This combined with an incentive to hold the least amount of capital possible, implies  $k = \phi s$ .<sup>17</sup>

The objective function (4) then becomes

$$\max_s p(s) \{(1-t)[s - i(1-\phi s) - c] + \phi s\} \frac{1}{1+R} - \phi s \quad (23)$$

The first order condition (FOC) after multiplication with  $1+R$  reads<sup>18</sup>

$$\begin{aligned} & p'(s) \{(1-t)[s - i(1-\phi s) - c] + \phi s\} \\ & + p(s) [(1-t)(1+i\phi) + \phi] - \phi(1+R) \\ & = 0 \end{aligned} \quad (24)$$

Rewriting the FOC, the SOC can then be expressed as (see Appendix C)

$$\Gamma(s) [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R)$$

The SOC certainly holds if the condition  $\Gamma(s) < 0$  applies and if  $p(s)$  is convex so that  $p''(s) > 0$ . Note that

$$\Gamma(s) [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) < 0 \quad (25)$$

is a weaker requirement if  $p(s)$  is convex and stronger if the success probability is concave. Throughout this section we maintain that this modified SOC does apply.

The effect of a change in the tax rate  $t$ , the capital requirement  $\phi$  and the embedded moral hazard follow from total differentiation of the FOC (24). See the Appendix C for derivations.

The embedded moral hazard is

$$\frac{ds}{dt} = - \frac{(1-t)\{-p'(s)(1-\phi s) + p(s)\phi\}}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}} > 0$$

provided that the modified SOC (25) is satisfied.

If the tax rate is increased we find

$$\frac{ds}{dt} = \frac{\{p'(s)[s - i(1-\phi s) - c] + p(s)(1+i\phi)\}}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}} < 0$$

<sup>17</sup> As long as  $1+R > p(s)[(1-t)i+1]$ , a bank prefers to operate with zero capital. This is an artifact the "cheap" debt relative to the cost of equity.

<sup>18</sup> Note that a necessary condition for the owners of the bank to participate is that at the optimum  $s$ , operating income should be positive, i.e.  $p(s)[s - i(1-\phi s) - c] > 0$ , as otherwise the owners expect a negative NPV and would close the bank.

After rewriting the numerator and given that  $(1 + R) - p'(s)s - p(s) > 0$ , one shows that the numerator is unambiguously positive.

Lastly, consider the effect of an increase of the Basel III capital requirement

$$\frac{ds}{d\phi} = \frac{-\{p'(s)[(1-t)is + s] + p(s)[(1-t)i + 1] - (1+R)\}}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}}$$

The numerator is positive, since by assumption  $-p'(s) > 0$ , by definition  $1 - p(s) \geq 0$  and as  $p(s)(1-t) < 1$  while investors typically require  $R > i$ , so that  $R > p(s)(1-t)i$ . It follows that  $ds/d\phi < 0$ . In the case of a binding leverage restriction, we already showed that an increase in the required amount of capital under the Basel I rules lowers risk taking. The current result claims that this effect is preserved under the newer Basel III risk based capital rules.

In summary, the same standard effects occur under the Basel III rules as under the Basel I rule. We are ready to investigate the effects of the alternative tax regimes, i.e., a bank levy, the ACE and the TCR under the new Basel rules.

#### 4.1 Bank levy

With a bank levy  $\lambda$ , the modified objective function of a bank becomes

$$\max_s p(s) \{(1-t)[s - (i + \lambda)(1 - \phi s) - c] + \phi s\} \frac{1}{1+R} - \phi s$$

Given the above and noting where  $\lambda$  enters the objective function, the FOC and modified SOC can be readily found respectively as

$$\begin{aligned} & p'(s) \{(1-t)[s - (i + \lambda)(1 - \phi s) - c] + \phi s\} \\ & + p(s)[(1-t)(1 + (i + \lambda)\phi) + \phi] - \phi(1+R) \\ & = 0 \end{aligned}$$

and

$$\Gamma(s)[(1-t)(1 + (i + \lambda)\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R) < 0 \quad (26)$$

Total differentiation of the FOC with respect to  $s$  and  $\lambda$  then gives

$$\frac{ds}{d\lambda} = -\frac{(1-t)\{-p'(s)(1 - \phi s) + p(s)\phi\}}{\Gamma(s)[(1-t)(1 + (i + \lambda)\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)}$$

Since the numerator is unambiguously positive, by the modified SOC (26), we have

**Proposition 5** *With the risk-based capital requirement  $k = \phi s$ , risk taking is increasing in the bank levy  $\lambda$ ,  $ds/d\lambda > 0$ .*

Thus we again find that a bank levy increases risk taking.

## 4.2 ACE with a risk based capital requirement

The ACE deduction is applied to the notional return on equity. At the same time the tax rate  $t$  is increased to finance this deduction to maintain bank profits. Let  $g$  denote the notional return on equity  $k$ , i.e.  $gk$ , which equals  $g\phi s$  if the Basel requirement binds. The deduction from taxes is imputed as a tax refund on capital  $tgk = tg\phi s$  in the good state. The objective function of the bank becomes (i.e., restate (13)),

$$\max_s p(s) \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} \frac{1}{1+R} - \phi s \quad (27)$$

The FOC reads

$$\begin{aligned} & p'(s) \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} \\ & + p(s) \{(1-t)(1+i\phi) + (1+tg)\phi\} - \phi(1+R) \\ & = 0 \end{aligned} \quad (28)$$

This implies that at the optimum

$$\begin{aligned} & \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} \\ & = -\frac{p(s)}{p'(s)} \{(1-t)(1+i\phi) + (1+tg)\phi\} + \frac{\phi(1+R)}{p'(s)} \end{aligned}$$

One then verifies (see below) that the SOC holds if the following modified condition applies

$$\Gamma(s) \{(1-t)(1+i\phi) + (1+tg)\phi\} + p''(s)\phi(1+R)/p'(s) < 0 \quad (29)$$

This is similar to the modified SOC (25) above.

We consider changes in the notional rate  $g$  and tax rate  $t$  in such a way that at the margin the changes in the tax bill are neutral for the bank. For this to hold across the banking sector, banks need to be homogenous and hence we deal with the 'representative' bank. In the Appendix C we prove the following claim:

**Proposition 6 (ACE regime)** *Given a risk based capital restriction, the introduction of a notional return on equity deductible  $g$  compensated by a change in the corporate tax rate  $t$  reduces risk taking incentives if the modified SOC (29) holds:*

$$\frac{ds}{dt} \Big|_{profits} = \frac{p(s) \frac{i+c}{s}}{\Gamma(s) \{(1-t)(1+i\phi) + (1+tg)\phi\} + \frac{p''(s)}{p'(s)} \phi(1+R)} < 0$$

## 4.3 TCR

We turn to the TCR regime combined with the restriction on capital. Suppose a binding constraint  $\beta$ ,  $\beta < 1$ , is introduced such that only  $\beta i(1-k) = \beta i(1-\phi s)$

of the deposit interest expenses are tax deductible. We can restate the objective function for TCR combined with the capital rule as

$$\max_s p(s) \{(1-t)[s-c] - i(1-\phi s) + t\beta(1-\phi s)i + \phi s\} \frac{1}{1+R} - \phi s \quad (30)$$

The FOC becomes

$$\begin{aligned} & p'(s) \{(1-t)[s-c] - (1-\phi s)(1-t\beta)i + \phi s\} \\ & + p(s) [(1-t) + \phi(1-t\beta)i + \phi] - \phi(1+R) \\ & = 0 \end{aligned} \quad (31)$$

Which can be re-expressed as

$$\begin{aligned} & \{(1-t)[s-c] - (1-\phi s)(1-t\beta)i + \phi s\} \\ & = -\frac{p(s)}{p'(s)} [(1-t) + \phi(1-t\beta)i + \phi] + \frac{\phi}{p'(s)} (1+R) \end{aligned}$$

The modified SOC for a maximum is

$$\Gamma(s) [(1-t) + \phi(1-t\beta)i + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) < 0 \quad (32)$$

and holds provided that  $\Gamma(s) < 0$  and that the second part does not upset this (for which convexity of  $p(s)$  suffices).

Suppose that changes in the cap rate on interest deductibility are executed in such a way that bank profits are unaffected. In the Appendix C we prove:

**Proposition 7** *The TCR regime with a risk based capital requirement increases risk taking incentives since*

$$\frac{ds}{dt} \bigg|_{profits} = \frac{p(s)^{\frac{1-\phi c}{1-\phi s}}}{\Gamma(s) [(1-t) + \phi(1-t\beta)i + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R)} < 0 \quad (33)$$

Note that  $1 > \phi s$  since the bank is only partially funded by capital and it stands to reason that  $1 - \phi c > 0$ . Hence, an increase in the cap, i.e., lowering  $\beta$  and lowering  $t$  in such a way so as to maintain bank profits, unambiguously increases risk taking by the bank, even in the presence of binding risk based capital regulation. The intuition is that the cap on the interest deductibility increases the (net) interest burden in the good state, and this can be reduced in expected value sense by increasing risk. The commensurate tax rate reduction also increases risk taking incentives as a smaller proportion of the returns  $s$  is taxed away.

To summarize, in the case Basel III risk based capital requirements ( $k = \phi s$ ), a bank levy and the TCR scheme both increase risk taking, while the ACE lowers risk. These results resemble those obtained earlier when capital was not risk-based.

## 5 Conclusion

In this paper we investigate the incentives for risk taking by banks under alternative corporate tax regimes and where leverage constraints are binding. Bank levies on debt financing have straightforward effects on risk taking: asset risk increases. Capping interest deductions (also called the thin capitalization rule: TCR) has similar effects, while introducing a tax advantage of equity financing (the allowance for corporate equity: ACE) reduces risk taking. These effects show that from the perspective of containing risk ACE is superior. Indeed, if the two regimes ACE and TCR are compared at equal expected bank profits, so that banks are indifferent, the ACE is risk reducing, while the TCR enhances risk taking.

If, however, the question is how to increase tax revenues from banks without affecting their asset risk, both the TCR and ACE can work. TCR works possibly most easily. Because increasing the corporate tax rate together with introducing the TCR cap both enhance government revenues, yet have opposing effects on banks risk taking. With ACE a more delicate calibration is needed.

We have focussed on the tax issues related to risk taking by banks. In a broader context there are other issues to consider, like the positive externality of the maintenance of the payment system and the effect of risk choices by banks on the level of innovation by firms, and the wellbeing of society at large. Furthermore, changes in the tax regime may also have an effect of the required rate of return by equity holders and systemic risk in the banking sector. We have left these interesting general equilibrium issues aside for future research.

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## 6 Appendix

### 6.1 Appendix A - Condition 1

In the main text we claimed that the Condition 1 is implied by the weak assumption that the distribution is in the max-domain of attraction of any one of the three extreme value distribution. This is made precise in the following result:

**Result 1** *Suppose that a distribution  $F(x)$  is in the max-domain of attraction of one of the three extreme value distributions for the maximum and satisfies one of the three sufficient von Mises conditions for this to be the case. Assume furthermore that the distribution is continuous, is twice differentiable and has a bounded first moment. Then it holds*

$$-\frac{p(s)p''(s)}{p'(s)} + 2p'(s) < 0$$

for  $s$  sufficiently large.

**Proof.** Note that the condition in terms of the distribution function  $F(x)$  and its derivatives  $f(x)$ ,  $f'(x)$  can be stated as follows

$$-\frac{[1 - F(x)][-f'(x)]}{[-f(x)]} + 2[-f(x)] < 0 \quad (34)$$

i/ The sufficient von Mises condition for a distribution to be in the domain of attraction of the Gumbel limit distribution  $\exp(-e^{-x})$  reads

$$\lim_{t \uparrow q} \frac{f'(t)[1 - F(t)]}{f(t)^2} = -1$$

where  $q \leq \infty$  and  $f'(t) < 0$  on some interval  $(t_0, q)$ . Rewriting (34), we get for sufficiently large  $x$

$$\begin{aligned} & \lim_{t \uparrow q} [-f(x)] \frac{[1 - F(x)][f'(x)]}{[-f(x)]^2} + 2[-f(x)] \\ &= [-f(x)] \times (-1) - 2f(x) = -f(x) < 0 \end{aligned}$$

ii/ The sufficient von Mises condition for a distribution to be in the domain of attraction of the Fréchet limit distribution  $\exp(-x^{-1/\gamma})$  reads

$$\lim_{t \rightarrow \infty} \frac{tf(t)}{[1 - F(t)]} = \frac{1}{\gamma}$$

Rewriting (34) again, we get

$$\begin{aligned} -\frac{[1 - F(x)][-f'(x)]}{[-f(x)]} + 2[-f(x)] &= \frac{[1 - F(x)]}{xf(x)} [-xf'(x)] - 2f(x) \\ &\sim \gamma [-xf'(x)] - 2f(x) \end{aligned}$$

for  $x$  large and where " $\sim$ " means that the ratio of the left hand side to the right hand side tends to 1 for  $x$  tending to infinity. Apply l'Hospital's rule to the von Mises condition

$$\lim_{x \rightarrow \infty} \frac{xf(x)}{[1 - F(x)]} = \lim_{x \rightarrow \infty} \frac{f(x) + xf'(x)}{-f(x)} = \frac{1}{\gamma}$$

Hence

$$-xf'(x) \sim \left(\frac{1}{\gamma} + 1\right) f(x)$$

Thus

$$\begin{aligned} \gamma[-xf'(x)] - 2f(x) &\sim (\gamma + 1)f(x) - 2f(x) \\ &= (\gamma - 1)f(x) \end{aligned}$$

which is negative as long as  $1 > \gamma$ . This corresponds to the requirement that the mean is bounded as  $1/\gamma$  is one to one with the number of bounded moments.

iii/ The sufficient von Mises condition for a distribution to be in the domain of attraction of the Weibull limit distribution  $\exp(-(-x)^\eta)$  is

$$\lim_{t \uparrow q} \frac{(q-t)f(t)}{[1 - F(t)]} = \eta$$

where  $q < \infty$  on some finite interval  $(t_0, q)$ . Apply l'Hospital's rule to the von Mises condition

$$\lim_{x \uparrow q} \frac{(q-x)f(x)}{[1 - F(x)]} = \lim_{x \uparrow q} \frac{-f(x) + (q-x)f'(x)}{-f(x)} = \eta$$

Thus for  $x$  close to the (finite) endpoint  $q$

$$-\frac{1}{\eta}(q-x)f'(x) \sim \frac{\eta-1}{\eta}f$$

Hence, by the von Mises condition

$$\begin{aligned} \frac{[1 - F(x)]}{f(x)}[-f'(x)] - 2f(x) &\sim \frac{q-x}{\eta}[-f'(x)] - 2f(x) \\ &\sim \frac{\eta-1}{\eta}f(x) - 2f(x) \\ &= -\left(\frac{1}{\eta} + 1\right)f(x) < 0 \end{aligned}$$

## 6.2 Appendix B

This Appendix contains the proofs of the results in sections 2 and 3 not presented in the main text. We start with the results for the bank levy from section 2.

### 6.2.1 bank levy analysis

The modified objective function including the bank levy as discussed in the main text is

$$\max_s p(s) \{(1-t)[s - i(1-k) - \lambda(1-k) - c(s)] + k\} \frac{1}{1+R} - k$$

The FOC is

$$p'(s) \{(1-t)[s - i(1-k) - \lambda(1-k) - c(s)] + k\} + p(s)(1-t)(1-c'(s)) = 0$$

By differentiating the FOC with respect to  $s$  and  $\lambda$ , one obtains

$$\begin{aligned} & p''(s) \{(1-t)[s - i(1-k) - \lambda(1-k) - c(s)] + k\} ds \\ & + [2p'(s)(1-t)(1-c'(s)) - p(s)(1-t)c''(s)] ds \\ & = p'(s)(1-t)(1-k) d\lambda \end{aligned}$$

Use the FOC and the properties of the fair premium in (2) and (3) to simplify this total differential

$$\Gamma(s) ds = p'(s)(1-k) d\lambda$$

and obtain

$$\frac{ds}{d\lambda} = \frac{p'(s)}{\Gamma(s)} (1-k) > 0$$

### 6.2.2 Proof of Corollary 2, section 3

Use (19). The FOC reads

$$p'(s) \{(1-t)[s - c(s)] - i(1-k) + k + tg\} + (1-t)p(s)[1 - c'(s)] = 0$$

One shows that the SOC is satisfied if Condition 1 applies.

Total differentiation of the FOC yields

$$\begin{aligned} & (1-t)\Gamma(s) ds \\ & = \{p'(s)(s - c(s)) - p'(s)g + p(s)[1 - c'(s)]\} dt + [-tp'(s)] dg \end{aligned}$$

Consider changes in  $t$  and  $g$  such that bank profits do not change. Again by the envelope theorem, using (19), this requires

$$-(s - c(s)) dt + gdt + t dg = 0$$

so that

$$\frac{dt}{dg} = \frac{t}{(s - c(s)) - g} \quad (35)$$

Simplifying the above total differential using (35) gives

$$\begin{aligned} & (1-t)\Gamma(s) ds \\ & = [p'(s)(s - c(s)) - p'(s)g + p(s)[1 - c'(s)]] dt + [-tp'(s)] dg \\ & = p(s)[1 - c'(s)] dt \end{aligned}$$

Thus we have

$$\frac{ds}{dt} \Big|_{\text{neutrality}} = \frac{p(s)[1 - c'(s)]}{(1-t)\Gamma(s)} < 0$$

Since  $c'(s) < 1$  by the FOC.

### 6.2.3 Proof of Proposition 4 and Lemma 3, section 3

Recall the objective function

$$\max_s p(s) \{(1-t)[s - c(s)] - i(1-k) + t\beta(1-k)i + k\} \frac{1}{1+R} - k$$

The FOC then reads

$$\begin{aligned} 0 &= p'(s) \{(1-t)[s - c(s)] - (1-k)(1-t\beta)i + k\} \\ &\quad + (1-t)p(s)[1 - c'(s)] \end{aligned} \quad (36)$$

Which implies

$$s - c(s) - \frac{(1-k)(1-t\beta)i - k}{1-t} = -\frac{p(s)}{p'(s)}[1 - c'(s)] \quad (37)$$

The SOC for a maximum is

$$\begin{aligned} &p''(s) \{(1-t)[s - c(s)] - (1-k)(1-t\beta)i + k\} \\ &\quad + 2(1-t)p'(s)[1 - c'(s)] - (1-t)p(s)c''(s) \\ &= \Gamma(s)(1-t) < 0 \end{aligned}$$

where we use (37), and the derivatives of the fair premium function in (2), (3) as in the case of the ACE analysis.

We first prove the Proposition 4. Consider marginal changes in the cap  $\beta$  and the corporate tax rate  $t$  while keeping bank profits constant. To this end apply the envelope theorem. That is, differentiate (20) partially with respect  $t$  and  $\beta$  and equate the total effect to zero

$$p(s) \{[-[s - c] + \beta(1-k)i]dt + t(1-k)id\beta\} \frac{1}{1+R} = 0$$

Solve for  $d\beta/dt$

$$\frac{d\beta}{dt} = \frac{[s - c(s)] - i(1-k)\beta}{i(1-k)t} \quad (38)$$

Since  $s - c(s) - i(1-k) > 0$ , it follows immediately that if  $\beta$  is lowered, the tax rate  $t$  has to be lowered as well to maintain the bank's level of profits.

To obtain the effects of a changes in the tax rate  $t$  and the cap  $\beta$ , differentiate the first order condition (36) with respect to  $s$ ,  $t$  and  $\beta$ :

$$\begin{aligned} &\Gamma(s)(1-t)ds + p'(s)(1-k)tid\beta \\ &= -\{p'(s)\{[-[s - c(s)] + (1-k)\beta i] - p(s)[1 - c'(s)]\}dt \end{aligned} \quad (39)$$

Combine the right hand side with (38) to obtain

$$\begin{aligned}
 & \Gamma(s)(1-t)ds \\
 = & \{p'(s)[s - c(s)] - p'(s)i(1-k)\beta + p(s)[1 - c'(s)]\}dt \\
 & - p'(s)i(1-k)t\left[\frac{s - c(s)}{i(1-k)t} - \frac{\beta}{t}\right]dt \\
 = & p(s)[1 - c'(s)]dt
 \end{aligned}$$

The claim

$$\frac{ds}{dt} \Big|_{\text{profits}} = \frac{p(s)[1 - c'(s)]}{\Gamma(s)(1-t)} < 0$$

in Proposition 4 follows.

As for the Lemma 3, set  $dt = 0$  in (39). This immediately yields

$$\frac{ds}{d\beta} = -\frac{p'(s)(1-k)ti}{\Gamma(s)(1-t)}$$

#### 6.2.4 raising revenues

To study how the TCR regime can be used to raise revenues, while maintaining the risk level  $s$ , set  $ds$  equal to zero in (39) and solve for  $d\beta/dt$ . Use the FOC in (37) to simplify the resulting expression. This gives

$$\frac{d\beta}{dt} = \frac{(1-k)(1-\beta)i - k}{(1-k)(1-t)ti}$$

If the cap is close to being non-binding, i.e., if  $\beta$  is close to 1, then

$$(1-k)(1-\beta)i - k < 0$$

With an increase in the tax rate, the cap should be increased (lower  $\beta$ ) to keep the risk level  $s$  constant. With a stronger binding cap, i.e.,  $\beta$  is lower and so less interest expenses are deductible, the tax base is increased while at the same time the risk level can be held constant.

### 6.3 Appendix C: Proofs for the Risk Based Capital Restrictions

Suppose a bank is not free to choose its mix of capital and deposit funding, but is constrained by the Basel risk based capital rules. Given the relative simplicity of the model, we use a linear rule and assume that it is binding.

Thus suppose that

$$k \geq \phi s$$

If binding, the objective function (4) changes from

$$\max_s p(s) \{(1-t)[s - i(1-k) - c] + k\} \frac{1}{1+R} - k$$

into

$$\max_s p(s) \{(1-t)[s - i(1-\phi s) - c] + \phi s\} \frac{1}{1+R} - \phi s \quad (40)$$

The FOC after multiplication with  $1+R$  reads<sup>19</sup>

$$\begin{aligned} & p'(s) \{(1-t)[s - i(1-\phi s) - c] + \phi s\} \\ & + p(s) [(1-t)(1+i\phi) + \phi] - \phi(1+R) \\ & = 0 \end{aligned} \quad (41)$$

Rewriting the FOC gives

$$\{(1-t)[s - i(1-\phi s) - c] + \phi s\} = -\frac{p(s)}{p'(s)} [(1-t)(1+i\phi) + \phi] + \frac{\phi(1+R)}{p'(s)}$$

Using this rewritten FOC, the SOC can then be expressed as

$$\begin{aligned} & p''(s) \{(1-t)[s - i(1-\phi s) - c] + \phi s\} + 2p'(s) [(1-t)(1+i\phi) + \phi] \\ & = -\frac{p''(s)p(s)}{p'(s)} [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) \\ & \quad + 2p'(s) [(1-t)(1+i\phi) + \phi] \\ & = \left[ -\frac{p(s)p''(s)}{p'(s)} + 2p'(s) \right] [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) \\ & = \Gamma(s) [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) \end{aligned}$$

The SOC certainly holds if  $\Gamma(s) < 0$  applies and if  $p(s)$  is convex so that  $p''(s) > 0$ . Note that

$$\Gamma(s) [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) < 0 \quad (42)$$

is a weaker requirement if  $p(s)$  is convex and stronger if the success probability is concave. Throughout this section we maintain that this modified SOC does apply.

The effect of a change in the tax rate  $t$ , the capital requirement  $\phi$  and the embedded moral hazard follow from total differentiation of the FOC:

$$\begin{aligned} & \left\{ \Gamma(s) [(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) \right\} ds \\ & + \{-p'(s)[s - i(1-\phi s) - c] - p(s)(1+i\phi)\} dt \\ & + \{p'(s)[(1-t)i s + s] + p(s)[(1-t)i + 1] - (1+R)\} d\phi \\ & + \{-p'(s)(1-t)(1-\phi s) + p(s)(1-t)\phi\} di \\ & = 0 \end{aligned}$$

<sup>19</sup>Note that a necessary condition for the owners of the bank to participate is that at the optimum  $s$ , expected operating income should be positive, i.e.  $p(s)[s - i(1-\phi s) - c] > 0$ , as otherwise the owners do expect a negative NPV and would close the bank.

The embedded moral hazard follows from (after total differentiation of the FOC (41))

$$\frac{ds}{di} = -\frac{(1-t)\{-p'(s)(1-\phi s) + p(s)\phi\}}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}} > 0$$

provided that the modified SOC (25) is satisfied.

If the tax rate is increased we get

$$\frac{ds}{dt} = \frac{\{p'(s)[s - i(1 - \phi s) - c] + p(s)(1 + i\phi)\}}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}}$$

Rewriting the FOC gives

$$\begin{aligned} & \phi(1+R) \\ = & p'(s)\{(1-t)[s - i(1 - \phi s) - c] + \phi s\} + p(s)[(1-t)(1+i\phi) + \phi] \\ = & p'(s)(1-t)[s - i(1 - \phi s) - c] + p'(s)\phi s + p(s)(1-t)(1+i\phi) + p(s)\phi \\ = & (1-t)\{p'(s)[s - i(1 - \phi s) - c] + p(s)(1+i\phi)\} \\ & + p'(s)\phi s + p(s)\phi \end{aligned}$$

or

$$\frac{\phi(1+R) - p'(s)\phi s - p(s)\phi}{1-t} = p'(s)[s - i(1 - \phi s) - c] + p(s)(1+i\phi)$$

so we need for  $ds/dt < 0$  that

$$(1+R) - p'(s)s - p(s) > 0$$

which should hold as

$$1 - p(s) \geq 0$$

and  $p'(s) < 0$ . Thus if the SOC (42) is satisfied, then  $ds/dt < 0$ .

Lastly, consider the effect of an increase of the Basel III capital requirement

$$\frac{ds}{d\phi} = \frac{-\{p'(s)[(1-t)is + s] + p(s)[(1-t)i + 1] - (1+R)\}}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}}$$

Consider the numerator, which can be rearranged as follows:

$$\begin{aligned} & -\{p'(s)[(1-t)is + s] + p(s)[(1-t)i + 1] - (1+R)\} \\ = & -p'(s)[(1-t)is + s] - p(s)[(1-t)i + 1] + (1+R) \\ = & -p'(s)[(1-t)is + s] + (1 - p(s)) + (R - p(s)(1-t)i) \end{aligned}$$

Note that by assumption  $-p'(s) > 0$ , by definition  $1 - p(s) \geq 0$  and as  $p(s)(1-t) < 1$  while investors typically require  $R > i$ , and so

$$R > p(s)(1-t)i$$

We have obtained the result from the main text

$$\frac{ds}{d\phi} = \frac{-p'(s)[(1-t)is + s] + (1 - p(s)) + (R - p(s)(1-t)i)}{\left\{\Gamma(s)[(1-t)(1+i\phi) + \phi] + \frac{p''(s)}{p'(s)}\phi(1+R)\right\}} < 0$$

### 6.3.1 ACE; proof of Proposition 6

The ACE deduction is applied to the notional return on equity. At the same time the tax rate  $t$  is increased to compensate for the loss in revenue in such a way that bank profits are not affected. Let  $g$  denote the notional return on equity  $k$ , i.e.  $gk$ , which equals  $g\phi s$  if the Basel requirement binds. The deduction from taxes is imputed as a tax refund on capital  $tgk = tg\phi s$  in the good state. The objective function of the bank becomes (i.e. restate (13)),

$$\max_s p(s) \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} \frac{1}{1+R} - \phi s \quad (43)$$

The FOC reads

$$\begin{aligned} & p'(s) \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} \\ & + p(s) \{(1-t)(1+i\phi) + (1+tg)\phi\} - \phi(1+R) \\ & = 0 \end{aligned} \quad (44)$$

This implies that at the optimum

$$\begin{aligned} & \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} \\ & = -\frac{p(s)}{p'(s)} \{(1-t)(1+i\phi) + (1+tg)\phi\} + \frac{\phi(1+R)}{p'(s)} \end{aligned}$$

One then verifies (see below) that the SOC holds if the following modified condition applies

$$\Gamma(s) \{(1-t)(1+i\phi) + (1+tg)\phi\} + p''(s)\phi(1+R)/p'(s) < 0 \quad (45)$$

This is quite similar to the modified SOC (25) in the main text.

We consider changes in the notional rate  $g$  and tax rate  $t$  without changing expected bank profits. Use the envelope theorem. Differentiate (43) partially with respect  $g$  and  $t$ , and equate the sum of the partial derivatives to zero:

$$p(s) \{-[s - i(1-\phi s) - c] dt + g\phi s dt + t\phi s dg\} \frac{1}{1+R} = 0$$

Solve for  $dt/dg$ :

$$\frac{dt}{dg} = \frac{t\phi s}{[s - i(1-\phi s) - c] - g\phi s} \quad (46)$$

We are ready to investigate the effects of introducing a notional return on equity accompanied by a change in the corporate income tax rate. To this end differentiate the first order condition (44) totally with respect to  $s$ ,  $g$  and  $t$

$$\begin{aligned} 0 = & \\ & p''(s) \{(1-t)[s - i(1-\phi s) - c] + (1+tg)\phi s\} ds \\ & + 2p'(s) \{(1-t)(1+i\phi) + (1+tg)\phi\} ds \\ & + \{-p'(s)[s - i(1-\phi s) - c] + p'(s)g\phi s + p(s)[- (1+i\phi) + g\phi]\} dt \\ & + \{p'(s)t\phi s + p(s)t\phi\} dg \end{aligned}$$

Use the rewritten FOC and (46) to simplify the resulting expression

$$\begin{aligned}
& \left[ \Gamma(s) \{ (1-t)(1+i\phi) + (1+tg)\phi \} + p''(s) \frac{\phi(1+R)}{p'(s)} \right] ds \\
&= -\{ -p'(s) [s - i(1-\phi s) - c] + p'(s) g\phi s + p(s) [-(1+i\phi) + g\phi] \} dt \\
&\quad - \{ p'(s) t\phi s + p(s) t\phi \} \frac{[s - i(1-\phi s) - c] - g\phi s}{t\phi s} dt \\
&= -\{ p(s) [-(1+i\phi) + g\phi] \} dt - p(s) t\phi \frac{[s - i(1-\phi s) - c] - g\phi s}{t\phi s} dt \\
&= -p(s) \left\{ -(1+i\phi) + g\phi + 1 - g\phi + i\phi + \frac{-i - c}{s} \right\} dt \\
&= p(s) \frac{i + c}{s} dt
\end{aligned}$$

Thus with constant profits

$$\begin{aligned}
& \left. \frac{ds}{dt} \right|_{\text{profits}} \quad (47) \\
&= \frac{p(s) \frac{i+c}{s}}{\Gamma(s) \{ (1-t)(1+i\phi) + (1+tg)\phi \} + p''(s) \phi(1+R) / p'(s)} < 0
\end{aligned}$$

### 6.3.2 TCR; proof of Proposition 7

We turn to the TCR regime combined with the restriction on capital. There is a binding constraint  $\beta$ ,  $\beta < 1$ , such that only  $\beta i(1-k) = \beta i(1-\phi s)$  of the deposit interest expenses are tax deductible. We can restate the objective function for TCR combined with the capital rule as

$$\max_s p(s) \{ (1-t)[s-c] - i(1-\phi s) + t\beta(1-\phi s)i + \phi s \} \frac{1}{1+R} - \phi s \quad (48)$$

The FOC becomes

$$\begin{aligned}
& p'(s) \{ (1-t)[s-c] - (1-\phi s)(1-t\beta)i + \phi s \} \\
&+ p(s) [(1-t) + \phi(1-t\beta)i + \phi] - \phi(1+R) \\
&= 0 \quad (49)
\end{aligned}$$

Which can be re-expressed as

$$\begin{aligned}
& \{ (1-t)[s-c] - (1-\phi s)(1-t\beta)i + \phi s \} \\
&= -\frac{p(s)}{p'(s)} [(1-t) + \phi(1-t\beta)i + \phi] + \frac{\phi}{p'(s)} (1+R)
\end{aligned}$$

The modified SOC for a maximum is

$$\Gamma(s) [(1-t) + \phi(1-t\beta)i + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) < 0 \quad (50)$$

and holds provided that  $\Gamma(s) < 0$  and that the second part does not upset this (for which convexity of  $p(s)$  suffices).

Suppose that changes in the cap rate on interest deductibility are executed in a profit neutral way. Invoking the envelope theorem, this requires

$$\{-(s-c) + \beta(1-\phi s)i\} dt + \{t(1-\phi s)i\} d\beta = 0$$

So that

$$\frac{d\beta}{dt} = -\frac{-(s-c) + \beta(1-\phi s)i}{t(1-\phi s)i}$$

Total differentiation of the FOC gives

$$\begin{aligned} 0 &= p''(s) \{(1-t)[s-c] - (1-\phi s)(1-t\beta)i + \phi s\} ds \\ &\quad + 2p'(s) [(1-t) + \phi(1-t\beta)i + \phi] ds \\ &\quad + \{p'(s) \{-(s-c) + (1-\phi s)\beta i\} + p(s) [-1 - \phi\beta i]\} dt \\ &\quad + \{p'(s)(1-\phi s)ti - p(s)\phi ti\} d\beta \end{aligned}$$

Using the rewritten FOC and  $d\beta/dt$  from above

$$\begin{aligned} &p''(s) \left\{ -\frac{p(s)}{p'(s)} [(1-t) + \phi(1-t\beta)i + \phi] + \frac{\phi}{p'(s)} (1+R) \right\} ds \\ &\quad + 2p'(s) [(1-t) + \phi(1-t\beta)i + \phi] ds \\ &= -\{p'(s) \{-(s-c) + (1-\phi s)\beta i\} + p(s) [-1 - \phi\beta i]\} dt \\ &\quad + \{p'(s)(1-\phi s)ti - p(s)\phi ti\} \frac{-(s-c) + \beta(1-\phi s)i}{t(1-\phi s)i} dt \end{aligned}$$

Simplify this expression

$$\begin{aligned} &\left\{ \Gamma(s) [(1-t) + \phi(1-t\beta)i + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R) \right\} ds \\ &= -\{p'(s) \{-(s-c) + (1-\phi s)\beta i\} + p(s) [-1 - \phi\beta i]\} dt \\ &\quad + p'(s) [-(s-c) + \beta(1-\phi s)i] dt - p(s) \phi \left[ \frac{-(s-c)}{(1-\phi s)} + \beta i \right] dt \\ &= p(s) [1 + \phi\beta i] dt - p(s) \phi \left[ \frac{-(s-c)}{(1-\phi s)} + \beta i \right] dt \\ &= p(s) \left[ 1 + \phi \frac{(s-c)}{(1-\phi s)} \right] dt \\ &= p(s) \frac{1 - \phi c}{1 - \phi s} dt \end{aligned}$$

Note that  $1 > \phi s$  since the bank is only partially funded by capital and it stands to reason that  $1 - \phi c > 0$ .

We have shown Proposition 7:

$$\frac{ds}{dt} \Big|_{\text{profits}} = \frac{p(s)^{\frac{1-\phi c}{1-\phi s}}}{\Gamma(s) [(1-t) + \phi(1-t\beta)i + \phi] + \frac{p''(s)}{p'(s)} \phi(1+R)} < 0 \quad (51)$$

**END**